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Microscopic conductivity of lattice fermions at equilibrium. I. Non-interacting particles

J. Math. Phys. **56**, 051901 (2015); <https://doi.org/10.1063/1.4919967>J.-B. Bru^{1, a)}, W. de Siqueira Pedra^{2, b)}, and  C. Hertling³[View Affiliations](#)[View Contributors](#)

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ABSTRACT

We consider free lattice fermions subjected to a static bounded potential and a time- and space-dependent electric field. For any bounded convex region $\mathcal{R} \subset \mathbb{R}^d$ ($d \geq 1$) of space, electric fields $\mathcal{E}$ within $\mathcal{R}$ drive currents. At leading order, uniformly with respect to the volume $|\mathcal{R}|$ of $\mathcal{R}$ and the particular choice of the static potential, the dependency on $\mathcal{E}$ of the current is linear and described by a conductivity (tempered, operator-valued) distribution. Because of the positivity of the heat production, the real part of its Fourier transform is a positive measure, named here (microscopic) conductivity measure of $\mathcal{R}$, in accordance with Ohm's law in Fourier space. This finite measure is the Fourier transform of a time-correlation function of current fluctuations, i.e., the conductivity distribution satisfies Green-Kubo relations. We additionally show that this measure can also be seen as the boundary value of the Laplace-Fourier transform of a so-called quantum current viscosity. The real and imaginary parts of conductivity distributions are related to each other via the Hilbert transform, i.e., they satisfy Kramers-Kronig relations. At leading order, uniformly with respect to parameters, the heat production is the classical work performed by electric fields on the system in presence of currents. The conductivity measure is uniformly bounded with respect to parameters of the system and it is never the trivial measure $0 \, d\nu$. Therefore, electric fields generally produce heat in such systems. In fact, the conductivity measure defines a quadratic form in the space of Schwartz functions, the Legendre-Fenchel transform of which describes the resistivity of the system. This leads to Joule's law, i.e., the heat produced by currents is proportional to the resistivity and the square of currents.



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